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Concept of Mathematics
Volume I & II
IIT - JEE (Main & Advanced) / XII / XI
ER. RAKESH KUMAR
 $\frac{0}{0}, \frac{\infty}{\infty}, 1^{\infty}, 0^0, 0 \times \infty, \infty - \infty, \infty^0 = ?$

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MATHEMATICS
Objective Questions

A BOOK BY ER. RAKESH KUMAR (NITIAN) (B.TECH-MECH, M.B.A-SCM)

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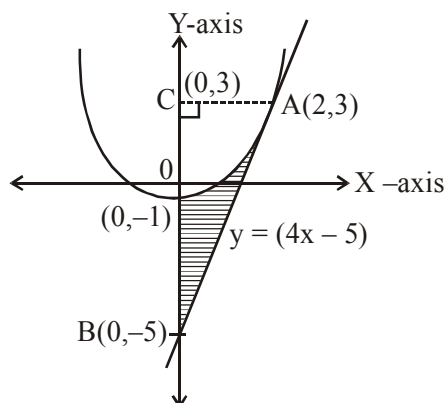
TEST PAPER OF JEE(MAIN) EXAMINATION – 2019
(Held On Wednesday 09th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM
MATHEMATICS

1. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

- (1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{8}{3}$ (4) $\frac{32}{3}$

Ans. (3)

Sol.



Equation of tangent at (2, 3) on $y = x^2 - 1$, is $y = (4x - 5)$ (i)

$\therefore$ Required shaded area

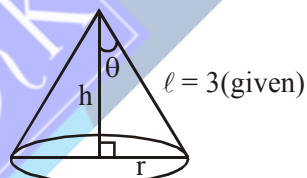
$$\begin{aligned} &= \text{ar}(\Delta ABC) - \int_{-1}^3 \sqrt{y+1} \, dy \\ &= \frac{1}{2} \cdot (8) \cdot (2) - \frac{2}{3} \left((y+1)^{3/2} \right)_{-1}^3 \\ &= 8 - \frac{16}{3} = \frac{8}{3} \text{ (square units)} \end{aligned}$$

2. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

- (1) $3\sqrt{3} \pi$ (2) 6π
 (3) $2\sqrt{3} \pi$ (4) $\frac{4}{3} \pi$

Ans. (3)

Sol.



$$\therefore h = 3 \cos \theta$$

$$r = 3 \sin \theta$$

Now,

$$V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} (9 \sin^2 \theta) \cdot (3 \cos \theta)$$

$$\therefore \frac{dV}{d\theta} = 0 \Rightarrow \sin \theta = \sqrt{\frac{2}{3}}$$

Also, $\left. \frac{d^2V}{d\theta^2} \right|_{\sin \theta = \sqrt{\frac{2}{3}}} = \text{negative}$

$\Rightarrow$ Volume is maximum,

when $\sin \theta = \sqrt{\frac{2}{3}}$

$$\therefore V_{\max} \left(\sin \theta = \sqrt{\frac{2}{3}} \right) = 2\sqrt{3} \pi \text{ (in cu. m)}$$

3. For $x^2 \neq n\pi + 1$, $n \in \mathbb{N}$ (the set of natural numbers), the integral

$$\int x \sqrt{\frac{2 \sin(x^2 - 1) - \sin 2(x^2 - 1)}{2 \sin(x^2 - 1) + \sin 2(x^2 - 1)}} dx$$

is equal to :

(where c is a constant of integration)

(1) $\log_e \left| \sec \left(\frac{x^2 - 1}{2} \right) \right| + c$

(2) $\log_e \left| \frac{1}{2} \sec^2(x^2 - 1) \right| + c$

(3) $\frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2 - 1}{2} \right) \right| + c$

(4) $\frac{1}{2} \log_e \left| \sec(x^2 - 1) \right| + c$

Ans. (1)

Sol. Put $(x^2 - 1) = t$

$$\Rightarrow 2x dx = dt$$

$$\therefore I = \frac{1}{2} \int \sqrt{\frac{1 - \cos t}{1 + \cos t}} dt$$

$$= \frac{1}{2} \int \tan \left(\frac{t}{2} \right) dt$$

$$= \ln \left| \sec \frac{t}{2} \right| + c$$

$$I = \ln \left| \sec \left(\frac{x^2 - 1}{2} \right) \right| + c$$

4. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :
 (1) 512 (2) -512 (3) -256 (4) 256

Ans. (3)

Sol. We have

$$(x + 1)^2 + 1 = 0$$

$$\Rightarrow (x + 1)^2 - (i)^2 = 0$$

$$\Rightarrow (x + 1 + i)(x + 1 - i) = 0$$

$$\therefore x = \underbrace{-(1+i)}_{\alpha(\text{let})} \quad \underbrace{-(1-i)}_{\beta(\text{let})}$$

$$\begin{aligned} \text{So, } \alpha^{15} + \beta^{15} &= (\alpha^2)^7 \alpha + (\beta^2)^7 \beta \\ &= -128(-i + 1 + i + 1) \\ &= -256 \end{aligned}$$

5. If $y = y(x)$ is the solution of the differential equation,

$$x \frac{dy}{dx} + 2y = x^2 \text{ satisfying}$$

$$y(1) = 1, \text{ then } y\left(\frac{1}{2}\right) \text{ is equal to :}$$

- (1) $\frac{7}{64}$ (2) $\frac{13}{16}$ (3) $\frac{49}{16}$ (4) $\frac{1}{4}$

Ans. (3)

Sol. $\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$

$$\Rightarrow \text{I.F.} = x^2$$

$$\therefore yx^2 = \frac{x^4}{4} + \frac{3}{4} \quad (\text{As, } y(1) = 1)$$

$$\therefore y\left(x = \frac{1}{2}\right) = \frac{49}{16}$$

6. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

- (1) $2\sqrt{3}y = 12x + 1$
 (2) $2\sqrt{3}y = -x - 12$
 (3) $\sqrt{3}y = x + 3$
 (4) $\sqrt{3}y = 3x + 1$

Ans. (3)

Sol. Let equation of tangent to the parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m},$$

$$\Rightarrow m^2x - ym + 1 = 0 \text{ is tangent to } x^2 + y^2 - 6x = 0$$

$$\Rightarrow \frac{|3m^2 + 1|}{\sqrt{m^4 + m^2}} = 3$$

$$m = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \text{tangent are } x + \sqrt{3}y + 3 = 0$$

$$\text{and } x - \sqrt{3}y + 3 = 0$$

7. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:
 (1) 200 (2) 300 (3) 500 (4) 350

Ans. (2)

Sol. Required number of ways

= Total number of ways - When A and B are always included.

$$= {}^5C_2 \cdot {}^7C_3 - {}^5C_1 \cdot {}^5C_2 = 300$$

8. Three circles of radii a, b, c ($a < b < c$) touch each other externally. If they have x-axis as a common tangent, then :

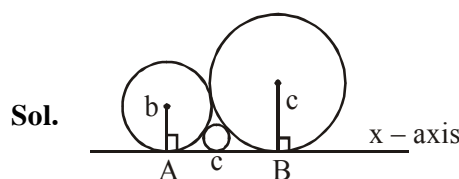
(1) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(2) a, b, c are in A. P.

(3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A. P.

(4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Ans. (1)



Sol.

$$AB = AC + CB$$

$$\sqrt{(b+c)^2 - (b-c)^2}$$

$$= \sqrt{(b+a)^2 - (b-a)^2} + \sqrt{(a+c)^2 - (a-c)^2}$$

$$\sqrt{bc} = \sqrt{ab} + \sqrt{ac}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$$

9. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

- (1) 14 (2) 6
(3) 4 (4) 8

Ans. (4)

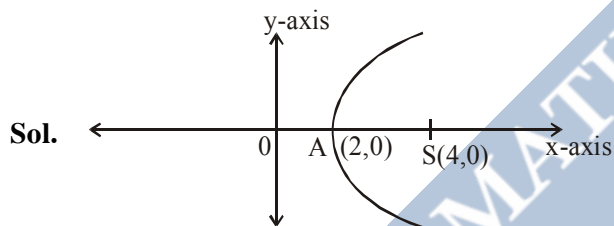
Sol. $\frac{2^{403}}{15} = \frac{2^3 \cdot (2^4)^{100}}{15} = \frac{8}{15} (15+1)^{100}$
 $= \frac{8}{15} (15\lambda + 1) = 8\lambda + \frac{8}{15}$
 $\therefore 8\lambda$ is integer

$\Rightarrow$ fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

10. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

- (1) (4, -4) (2) (5, $2\sqrt{6}$)
(3) (8, 6) (4) (6, $4\sqrt{2}$)

Ans. (3)



Sol.

equation of parabola is

$$y^2 = 8(x - 2)$$

(8, 6) does not lie on parabola.

11. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point :

- (1) (-3, 0, -1) (2) (3, 3, -1)
(3) (3, 2, 1) (4) (-3, 1, 1)

Ans. (3)

Sol. Equation of plane

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

$$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$$

dr's of normal of the plane are

$$1 + 2\lambda, 1 + 3\lambda, 1 - \lambda$$

Since plane is parallel to y - axis, $1 + 3\lambda = 0$

$$\Rightarrow \lambda = -1/3$$

So the equation of plane is

$$x + 4z - 7 = 0$$

Point (3, 2, 1) satisfies this equation

Hence Answer is (3)

12. If a, b and c be three distinct real numbers in G. P. and $a + b + c = xb$, then x cannot be :

- (1) 4 (2) -3 (3) -2 (4) 2

Ans. (4)

Sol. $\frac{b}{r}, b, br \rightarrow G.P.$ ($|r| \neq 1$)

given $a + b + c = xb$

$$\Rightarrow b/r + b + br = xb$$

$$\Rightarrow b = 0 \text{ (not possible)}$$

$$\text{or } 1 + r + \frac{1}{r} = x \Rightarrow x - 1 = r + \frac{1}{r}$$

$$\Rightarrow x - 1 > 2 \text{ or } x - 1 < -2$$

$$\Rightarrow x > 3 \text{ or } x < -1$$

So x can't be '2'

13. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true ?

- (1) The lines are all parallel.
(2) Each line passes through the origin.

- (3) The lines are not concurrent

The lines are concurrent at the point

(4) $\left(\frac{3}{4}, \frac{1}{2}\right)$

Ans. (4)

Sol. Given set of lines $px + qy + r = 0$

given condition $3p + 2q + 4r = 0$

$$\Rightarrow \frac{3}{4}p + \frac{1}{2}q + r = 0$$

$$\Rightarrow \text{All lines pass through a fixed point } \left(\frac{3}{4}, \frac{1}{2}\right).$$

14. The system of linear equations.

$$x + y + z = 2$$

$$2x + 3y + 2z = 5$$

$$2x + 3y + (a^2 - 1)z = a + 1$$

- (1) has infinitely many solutions for $a = 4$

- (2) is inconsistent when $|a| = \sqrt{3}$

- (3) is inconsistent when $a = 4$

- (4) has a unique solution for $|a| = \sqrt{3}$

Ans. (2)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix} = a^2 - 3$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 3 & 2 \\ a+1 & 3 & a^2 - 1 \end{vmatrix} = a^2 - a + 1$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 2 & a+1 & a^2 - 1 \end{vmatrix} = a^2 - 3$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 2 & 3 & a+1 \end{vmatrix} = a - 4$$

$$D = 0 \text{ at } |a| = \sqrt{3} \text{ but } D_3 = \pm\sqrt{3} - 4 \neq 0$$

So the system is Inconsistent for $|a| = \sqrt{3}$

- 15.** Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to :-

- (1) $\frac{19}{2}$ (2) 8 (3) $\frac{17}{2}$ (4) 9

Ans. (1)

Sol. $\vec{a} \times \vec{c} = -\vec{b}$
 $(\vec{a} \times \vec{c}) \times \vec{a} = -\vec{b} \times \vec{a}$
 $\Rightarrow (\vec{a} \times \vec{c}) \times \vec{a} = \vec{a} \times \vec{b}$
 $\Rightarrow (\vec{a} \cdot \vec{a})\vec{c} - (\vec{c} \cdot \vec{a})\vec{a} = \vec{a} \times \vec{b}$
 $\Rightarrow 2\vec{c} - 4\vec{a} = \vec{a} \times \vec{b}$
 Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = -\hat{i} - \hat{j} + 2\hat{k}$
 So, $2\vec{c} = 4\hat{i} - 4\hat{j} - \hat{i} - \hat{j} + 2\hat{k}$
 $= 3\hat{i} - 5\hat{j} + 2\hat{k}$
 $\Rightarrow \vec{c} = \frac{3}{2}\hat{i} - \frac{5}{2}\hat{j} + \hat{k}$
 $|\vec{c}| = \sqrt{\frac{9}{4} + \frac{25}{4} + 1} = \sqrt{\frac{38}{4}} = \sqrt{\frac{19}{2}}$
 $|\vec{c}|^2 = \frac{19}{2}$

- 16.** Let $a_1, a_2, \dots, a_{30}$ be an A. P., $S = \sum_{i=1}^{30} a_i$ and

$$T = \sum_{i=1}^{15} a_{(2i-1)}. \text{ If } a_5 = 27 \text{ and } S - 2T = 75, \text{ then } a_{10} \text{ is equal to :}$$

- (1) 57 (2) 47 (3) 42 (4) 52

Ans. (4)

Sol. $S = a_1 + a_2 + \dots + a_{30}$

$$S = \frac{30}{2} [a_1 + a_{30}]$$

$$S = 15(a_1 + a_{30}) = 15(a_1 + a_1 + 29d)$$

$$T = a_1 + a_3 + \dots + a_{29}$$

$$= (a_1) + (a_1 + 2d) + \dots + (a_1 + 28d)$$

$$= 15a_1 + 2d(1 + 2 + \dots + 14)$$

$$T = 15a_1 + 210d$$

$$\text{Now use } S - 2T = 75$$

$$\Rightarrow 15(2a_1 + 29d) - 2(15a_1 + 210d) = 75$$

$$\Rightarrow d = 5$$

$$\text{Given } a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$$

$$\text{Now } a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$$

- 17.** 5 students of a class have an average height 150 cm and variance 18 cm². A new student, whose height is 156 cm, joined them. The variance (in cm²) of the height of these six students is:

- (1) 22 (2) 20 (3) 16 (4) 18

Ans. (2)

Sol. Given $\bar{x} = \frac{\sum x_i}{5} = 150$

$$\Rightarrow \sum_{i=1}^5 x_i = 750 \quad \dots(i)$$

$$\frac{\sum x_i^2}{5} - (\bar{x})^2 = 18$$

$$\frac{\sum x_i^2}{5} - (150)^2 = 18$$

$$\sum x_i^2 = 112590 \quad \dots(ii)$$

Given height of new student

$$x_6 = 156$$

$$\text{Now, } \bar{x}_{\text{new}} = \frac{\sum_{i=1}^6 x_i}{6} = \frac{750 + 156}{6} = 151$$

$$\text{Also, New variance} = \frac{\sum_{i=1}^6 x_i^2}{6} - (\bar{x}_{\text{new}})^2$$

$$= \frac{112590 + (156)^2}{6} - (151)^2$$

$$= 22821 - 22801 = 20$$

18. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals :

- (1) 52/169 (2) 25/169
(3) 49/169 (4) 24/169

Ans. (2)

Sol. Two cards are drawn successively with replacement

4 Aces 48 Non Aces

$$P(x=1) = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{48C_1}{52C_1} + \frac{48C_1}{52C_1} \times \frac{{}^4C_1}{52C_1} = \frac{24}{169}$$

$$P(x=2) = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^4C_1}{52C_1} = \frac{1}{169}$$

$$P(x=1) + P(x=2) = \frac{25}{169}$$

19. For $x \in \mathbb{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$,

$f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three

given functions. If a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to :-

- (1) $f_3(x)$ (2) $f_1(x)$
(3) $f_2(x)$ (4) $\frac{1}{x} f_3(x)$

Ans. (1)

Sol. Given $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$

$$(f_2 \circ J \circ f_1)(x) = f_3(x)$$

$$f_2 \circ (J(f_1(x))) = f_3(x)$$

$$f_2 \circ \left(J\left(\frac{1}{x}\right) \right) = \frac{1}{1-x}$$

$$1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$$

$$J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x} = \frac{-x}{1-x} = \frac{x}{x-1}$$

$$\text{Now } x \rightarrow \frac{1}{x}$$

$$J(x) = \frac{\frac{1}{x}}{\frac{1}{x}-1} = \frac{1}{1-x} = f_3(x)$$

20. Let

$$A = \left\{ 0 \in \left(-\frac{\pi}{2}, \pi \right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary} \right\}$$

Then the sum of the elements in A is :

- (1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$
(3) $\frac{3\pi}{4}$ (4) π

Ans. (2)

Sol. Given $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely img

so real part becomes zero.

$$z = \left(\frac{3+2i\sin\theta}{1-2i\sin\theta} \right) \times \left(\frac{1+2i\sin\theta}{1+2i\sin\theta} \right)$$

$$z = \frac{(3-4\sin^2\theta) + i(8\sin\theta)}{1+4\sin^2\theta}$$

Now $\text{Re}(z) = 0$

$$\frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm \frac{\sqrt{3}}{2} \Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\therefore \theta \in \left(-\frac{\pi}{2}, \pi \right)$$

then sum of the elements in A is

$$-\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$$

21. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to :

- (1) 4/9 (2) 7/17
(3) 8/17 (4) 8/15

Ans. (4)

Sol. Point of intersection is $P(2,6)$.

$$\text{Also, } m_1 = \left(\frac{dy}{dx} \right)_{P(2,6)} = -2x = -4$$

$$m_2 = \left(\frac{dy}{dx} \right)_{P(2,6)} = 2x = 4$$

$$\therefore |\tan\theta| = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \frac{8}{15}$$

22. If $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then the matrix A^{-50}

when $\theta = \frac{\pi}{12}$, is equal to :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Ans. (1)

Sol. Here, $AA^T = I$

$$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\text{Also, } A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$$

$$\therefore A^{-50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

23. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

- (1) (2, 3] (2) (3, ∞)
(3) (3/2, 2] (4) (1, 3/2]

Ans. (2)

Sol. $e = \sqrt{1 + \tan^2 \theta} = \sec \theta$

$$\text{As, } \sec \theta > 2 \Rightarrow \cos \theta < \frac{1}{2}$$

$$\Rightarrow \theta \in (60^\circ, 90^\circ)$$

$$\text{Now, } \ell(L.R) = \frac{2b^2}{a} = 2 \frac{(1 - \cos^2 \theta)}{\cos \theta}$$

$$= 2(\sec \theta - \cos \theta)$$

Which is strictly increasing, so

$$\ell(L.R) \in (3, \infty).$$

24. The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$

and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

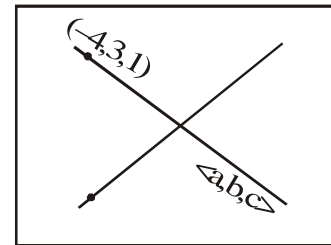
(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(3) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Ans. (2)



Sol.

Normal vector of plane containing two intersecting lines is parallel to vector.

$$(\vec{V}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

$$= -2\hat{i} + 6\hat{k}$$

$\therefore$ Required line is parallel to vector

$$(\vec{V}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\Rightarrow$ Required equation of line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

25. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

- (1) $13 - 4 \cos^6\theta$
 (2) $13 - 4 \cos^4\theta + 2 \sin^2\theta \cos^2\theta$
 (3) $13 - 4 \cos^2\theta + 6 \cos^4\theta$
 (4) $13 - 4 \cos^2\theta + 6 \sin^2\theta \cos^2\theta$

Ans. (1)

Sol. We have,

$$\begin{aligned} & 3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta \\ &= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta \\ &= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta \\ &= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3 \\ &= 13 - 4 \cos^6\theta \end{aligned}$$

26. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$ ($x > \frac{3}{4}$) then x is equal to :

- (1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$
 (3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

Ans. (1)

Sol. $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$ ($x > \frac{3}{4}$)

$$\cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3x}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right) = \sin^{-1}\left(\frac{2}{3x}\right)$$

$$\cos\left(\cos^{-1}\left(\frac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\left(\frac{2}{3x}\right)\right)$$

$$\frac{3}{4x} = \frac{\sqrt{9x^2 - 4}}{3x}$$

$$\frac{81}{16} + 4 = 9x^2$$

$$x^2 = \frac{145}{16 \times 9} \Rightarrow x = \frac{\sqrt{145}}{12}$$

27. The value of $\int_0^{\pi} |\cos x|^3 dx$

- (1) $2/3$ (2) 0
 (3) $-4/3$ (4) $4/3$

Ans. (4)

$$\begin{aligned} \text{Sol. } \int_0^{\pi} |\cos x|^3 dx &= \int_0^{\pi/2} \cos^3 x dx - \int_{\pi/2}^{\pi} \cos^3 x dx \\ &= \int_0^{\pi/2} \left(\frac{\cos 3x + 3\cos x}{4} \right) dx - \int_{\pi/2}^{\pi} \left(\frac{\cos 3x + 3\cos x}{4} \right) dx \\ &= \frac{1}{4} \left[\left(\frac{\sin 3x}{3} + 3\sin x \right) \right]_0^{\pi/2} - \left[\left(\frac{\sin 3x}{3} + 3\sin x \right) \right]_{\pi/2}^{\pi} \\ &= \frac{1}{4} \left[\left(\frac{-1}{3} + 3 \right) - (0 + 0) - \left\{ (0 + 0) - \left(\frac{-1}{3} + 3 \right) \right\} \right] \\ &= \frac{4}{3} \end{aligned}$$

28. If the Boolean expression

$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is:

- (1) $(\wedge, \vee)$
 (2) $(\vee, \vee)$
 (3) $(\wedge, \wedge)$
 (4) $(\vee, \wedge)$

Ans. (1)

Sol. $(p \oplus q) \wedge (\sim p \odot q) \equiv p \wedge q$ (given)

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$\sim p \vee q$	$\sim p \wedge q$	$(p \wedge q) \wedge (\sim p \vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

from truth table $(\oplus, \odot) = (\wedge, \vee)$

29. $\lim_{y \rightarrow 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

(1) exists and equals $\frac{1}{4\sqrt{2}}$

(2) does not exist

(3) exists and equals $\frac{1}{2\sqrt{2}}$

(4) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

Ans. (1)

Sol. $\lim_{y \rightarrow 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

$$= \lim_{y \rightarrow 0} \frac{1 + \sqrt{1+y^4} - 2}{y^4 \left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2} \right)}$$

$$= \lim_{y \rightarrow 0} \frac{(\sqrt{1+y^4} - 1)(\sqrt{1+y^4} + 1)}{y^4 \left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2} \right) (\sqrt{1+y^4} + 1)}$$

$$= \lim_{y \rightarrow 0} \frac{1+y^4-1}{y^4 \left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2} \right) (\sqrt{1+y^4} + 1)}$$

$$= \lim_{y \rightarrow 0} \frac{1}{\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2} \right) (\sqrt{1+y^4} + 1)} = \frac{1}{4\sqrt{2}}$$

30. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as :

$$f(x) = \begin{cases} 5, & \text{if } x \leq 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \leq x < 5 \\ 30, & \text{if } x \geq 5 \end{cases}$$

Then, f is :

(1) continuous if $a = 5$ and $b = 5$

(2) continuous if $a = -5$ and $b = 10$

(3) continuous if $a = 0$ and $b = 5$

(4) not continuous for any values of a and b

Ans. (4)

Sol. $f(x) = \begin{cases} 5 & \text{if } x \leq 1 \\ a + bx & \text{if } 1 < x < 3 \\ b + 5x & \text{if } 3 \leq x < 5 \\ 30 & \text{if } x \geq 5 \end{cases}$

$$f(1) = 5, \quad f(1^-) = 5, \quad f(1^+) = a + b$$

$$f(3^-) = a + 3b, \quad f(3) = b + 15, \quad f(3^+) = b + 15$$

$$f(5^-) = b + 25; \quad f(5) = 30 \quad f(5^+) = 30$$

from above we concluded that f is not continuous for any values of a and b .